Calculus

Name: Date:

Unit 1 Practice Test: Limits

Here's your chance to show what you know! You've got all you need in your brain, so trust yourself and put your calculator away. Make sure you show me all the cool work you can do to get your answer when appropriate.

1. Find the following limits if $f(x) = \frac{x^2+x-20}{x^2-16}$. DO NOT use "DNE" as an answer.

   a. $\lim_{x\to0} f(x) =$

   $\lim_{x\to0}\frac{x+5}{x+4} = \frac{0+5}{0+4} = \boxed{\frac{5}{4}}$

   $= \frac{(x+5)(x-4)}{(x+4)(x-4)}$

   $= \frac{x+5}{x+4} \quad x \neq 4$

   b. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{x^2+\cancel{x}-\cancel{20}}{x^2-\cancel{16}}$

   $\lim_{x\to\infty}\frac{x^2}{x^2} = \boxed{1}$

   c. $\lim_{x\to4} f(x) =$

   $\lim_{x\to4}\frac{(x+5)\cancel{(x-4)}}{(x+4)\cancel{(x-4)}} = \lim_{x\to4}\frac{(x+5)}{(x+4)} = \boxed{\frac{9}{8}}$

2. Graph and evaluate the following limits if $f(x) = \begin{cases} 1-2x & x \le -1 \\ \cancel{x+4} & x > -1 \end{cases}$

   $y = 1-2x$

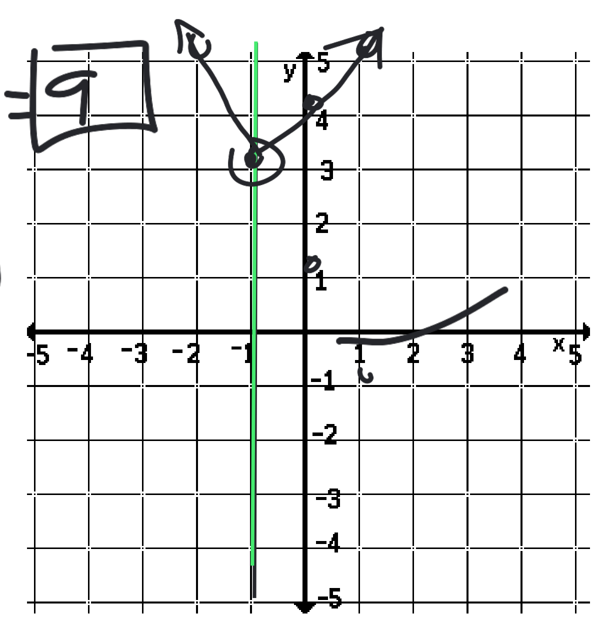
   a. $\lim_{x\to-4} f(x) = f(-4)$

   $= 1-2(-4) = \boxed{9}$

   b. $\lim_{x\to2} f(x) = f(2)$

   $= 2+4 = \boxed{6}$

   c. $\lim_{x\to-1} f(x) = \boxed{3}$



3. If $f(x) = \frac{ax-12}{3x-b}$, find the following…

   a. Evaluate the value of a if $\lim_{x\to\infty} f(x) = 5$.

   $\lim_{x\to\infty}\frac{ax-\cancel{12}}{3x-\cancel{b}} = \lim_{x\to\infty}\frac{ax}{3x} = \frac{a}{3} = 5$

   $\boxed{a=15}$

   b. Evaluate the value of b if $\lim_{x\to4} f(x)$ does not exists.

   $\lim_{x\to4}\frac{ax+12}{3x-b}$

   $3x-b=0$

   $3(4)-b=0$

   $\boxed{12=b}$

4. Create a single function with the following characteristics:
   - a removable discontinuity x = -3
   - a non-removable discontinuity at x = 2 (you don't need to simplify)

$$f(x)=\frac{(x+3)}{(x+3)(x-2)}$$

5. Evaluate the limits algebraically (a.k.a. analytically). Show all work; each intermediate step is potential points. Please circle your answers.

$(a+b)(a-b)=a^2-b^2$

a. $\lim_{x\to 2}\frac{x^2-4}{x^2+4x-12}=$

$$=\lim_{x\to 2}\frac{(x+2)\cancel{(x-2)}}{(x+6)\cancel{(x-2)}}$$

$$=\lim_{x\to 2}\frac{x+2}{x+6}=\frac{2+2}{2+6}=\frac{4}{8}=\boxed{\frac{1}{2}}$$

b. $\lim_{x\to -4}\frac{x+4}{3-\sqrt{5-x}}\cdot\frac{(3+\sqrt{5-x})}{(3+\sqrt{5-x})}$

$$=\lim_{x\to -4}\frac{(x+4)(3+\sqrt{5-x})}{9-(5-x)}$$

$$=\lim_{x\to -4}\frac{\cancel{(x+4)}(3+\sqrt{5-x})}{\cancel{(4+x)}}=3+\sqrt{5-(-4)}$$

$$=3+3=\boxed{6}$$

c. $\lim_{x\to 0}\cancel{\tan}\ \cot(3x)\sin(4x)=$

$$\lim_{x\to 0}\cot(3x)\sin(4x)$$

$$\lim_{x\to 0}\frac{\cos(3x)}{\sin(3x)}\cdot\sin 4x$$

$$\lim_{x\to 0}\frac{3x}{\sin(3x)}\cdot\frac{\sin(4x)}{4x}\cdot\frac{\cos(3x)\cdot 4x}{3x}$$

$$1\cdot 1\cdot\frac{1\cdot 4}{3}$$

$$=\boxed{\frac{4}{3}}$$

$$\lim_{x\to 0}\frac{\sin(3x)}{\cos(x)}\cdot\frac{\sin(4x)}{1}$$

$$\frac{\sin(3\cdot 0)}{\cos(0)}\cdot\frac{\sin(4\cdot 0)}{1}$$

$$\frac{0}{1}\cdot\frac{0}{1}=\boxed{0}$$

d. $\lim_{x\to 3}\frac{2-\frac{x+1}{x-1}}{x-3}\cdot\frac{(x-1)}{(x-1)}=$

$$=\lim_{x\to 3}\frac{2(x-1)-(x+1)}{(x-3)(x-1)}$$

$$=\lim_{x\to 3}\frac{\cancel{x-3}^{\,1}}{\cancel{(x-3)}(x-1)}$$

$$=\lim_{x\to 3}\frac{1}{x-1}=\frac{1}{3-1}=\boxed{\frac{1}{2}}$$

e. $\lim_{x\to\infty}\frac{4x^2}{(x^2-25)(x+1)}=$

$$=\lim_{x\to\infty}\frac{4x^2}{x^3+\cancel{x^2}-\cancel{25x}-\cancel{25}}$$

$$=\lim_{x\to\infty}\frac{4x^2}{\cancel{x^3}}=\lim_{x\to\infty}\frac{4}{x}$$

$$=\boxed{0}$$

f. $\lim_{x\to -\infty}\frac{\sqrt{49x^4+\cancel{\pi x-e^1}}}{(x^2-\cancel{25})}=$

$$\lim_{x\to -\infty}\frac{\sqrt{49x^4}}{x^2}=\lim_{x\to -\infty}\frac{7\cancel{x^2}}{\cancel{x^2}}=\boxed{7}$$

6. Use the graph of f(x) to evaluate the limits. If the limit does not exist, use DNE.

a. $\lim_{x \to -3} f(x) =$ 3

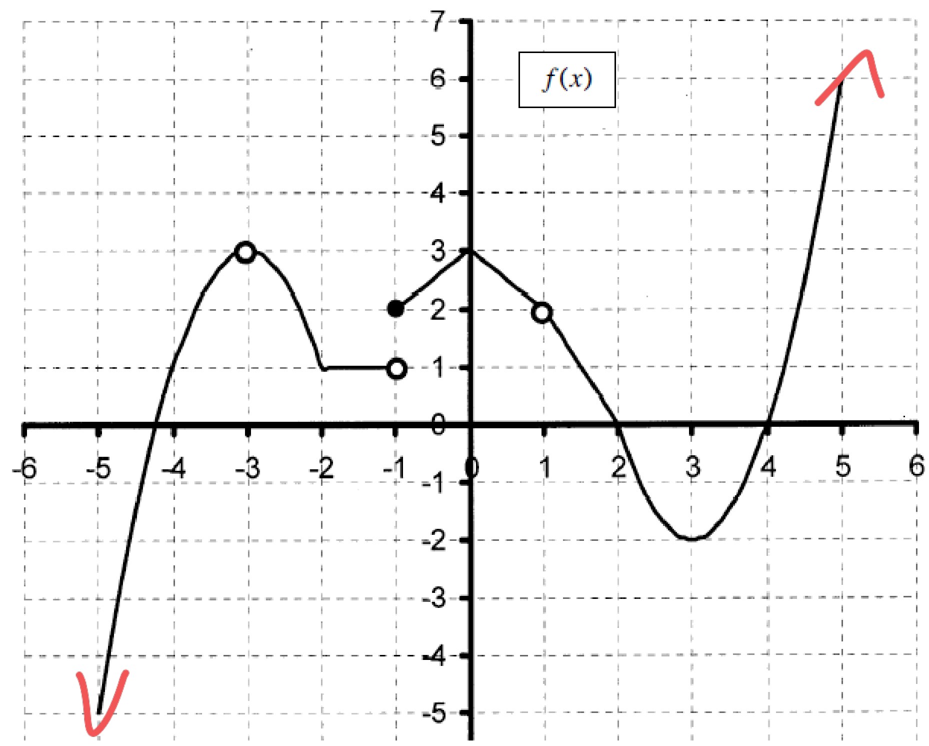
b. $\lim_{x \to 2} f(x) =$ 0

c. $\lim_{x \to -1^-} f(x) =$ 1

d. $\lim_{x \to -1^+} f(x) =$ 2

e. $\lim_{x \to -1} f(x) =$ DNE

f. $\lim_{x \to -\infty} f(x) =$ $-\infty$



7. The functions f and g are continuous for all real numbers. The table below give values of the functions at selected values of x. The function h is given by $h(x) = g(f(x)) + 2$.

x	$f(x)$	$g(x)$
1	3	4
3	9	-10
5	7	5
7	11	25

Since h(x) is continuous, and $h(1) < 0 < h(5)$, there is a w on $1 < w < 5$ such that $h(w) = 0$

Explain why there must be a value w for 1 < w < 5 such that h(w) = 0.

$h(1) = g(f(1)) + 2 = g(3) + 2 = -10 + 2 = -8$

$h(5) = g(f(5)) + 2 = g(7) + 2 = 25 + 2 = 27$

8. What are the two big questions that Calculus answers?

→ Tangent line Problem

→ Area under a curve